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SIMILARITY SOLUTIONS FOR MIXED CONVECTION FROM HORIZONTAL IMPERMEABLE SURFACES IN SATURATED POROUS MEDIA

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Abstract—Boundary-layer analysis is performed for mixed convection about a horizontal flat plate in a saturated porous medium with aiding external flows, where the governing parameter is found to be $Ra/(RePr)^{3/2}$. Similarity solutions are obtained for (i) horizontal flat plates at zero angle of attack with constant heat flux and (ii) stagnation point flows about horizontal flat plates with wall temperature varying as $T_w \propto x^2$. Temperature and velocity profiles for these two cases at selected values of $Ra/(RePr)^{3/2}$ are presented. The heat-transfer rate is shown to be asymptotically approaching the forced and free convection values as the value of $Ra/(RePr)^{3/2}$ approaches the limits of 0 and ∞. The criteria for pure and mixed convection about horizontal flat plates in porous media are established.

NOMENCLATURE

A, constant defined in equation (6a);
B, constant defined in equation (14b);
C, specific heat of the convective fluid;
f, dimensionless stream function defined by equation (16);
Gr, local Grashof number, $Gr \equiv g|T_w - T_\infty|\beta K x/v^2$;
g, acceleration due to gravity;
h, local heat-transfer coefficient;
K, permeability of the porous medium;
k_m, thermal conductivity of the saturated porous medium;
m, constant defined in equation (14b);
n, porosity;
Nu, local Nusselt number, $Nu = hx/k_m$;
p, pressure;
Pr, Prandtl number, $Pr \equiv v/\alpha$;
q, local heat-transfer rate;
Ra, modified local Rayleigh number, $Ra \equiv \rho_\infty g\beta K|T_w - T_\infty|x/\mu\alpha$;
Re, local Reynolds number, $Re \equiv U_\infty x/v$;
T, temperature;
U_∞, Darcy's velocity in x-direction outside the boundary layer;
u, Darcy's velocity in x-direction;
v, Darcy's velocity in y-direction;
x, coordinate in the horizontal direction;
y, coordinate in the vertical direction.

Greek symbols

α, equivalent thermal diffusivity;
β, coefficient of thermal expansion;
δ_T, thermal boundary-layer thickness;
η, dimensionless similarity variable defined in equation (15);
η_T, value of η at the edge of the thermal boundary layer;
θ, dimensionless temperature defined by equation (17);
λ, constant defined in equation (6a);
μ, viscosity of convective fluid;
v, kinematic viscosity of the convective fluid;
ρ, density of convective fluid;
ϕ, velocity potential;
ψ, stream function.

Subscripts

∞, condition at infinity;
f, convective fluid;
s, unsaturated porous medium;
w, condition at the wall.

INTRODUCTION

THE STUDY of mixed (combined free and forced) convection boundary-layer flows in a viscous fluid has received much attention in the past two decades (see Gebhart [1] for a review of the literature). Most of the analyses for mixed convection about inclined surfaces neglect the component of the buoyancy force normal to the surface. This approximation will break down completely when the inclined surface becomes horizontal where the buoyancy force is acting perpendicular to the surface. Thus, mixed convection about horizontal surfaces have been treated separately from those of inclined surfaces. Although similarity solutions have been obtained for mixed convection about inclined surfaces in a viscous fluid (Sparrow *et al.* [2]), they do not exist for mixed convection about horizontal surfaces where series solutions have been obtained instead [3–5].

The analogous problems of mixed convection in a porous medium have important applications in geothermal reservoirs where pressure gradients may be

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generated either by artificial withdrawal or injection of fluids or by natural recharge or discharge of meteoric water. Recently, a number of papers [6–11] have appeared on the study of mixed convection in a porous medium. In particular, the problem of mixed convection about inclined surfaces is considered by Cheng [11] who neglects the normal component of buoyancy force, and obtains similarity solutions for the special case where the free stream velocity and wall temperature vary according to the same power function of distance.

In this paper, we shall study mixed convection about horizontal surfaces embedded in a porous medium where gravitational force acts perpendicular to the surface. Similarity solutions are obtained for aiding flows over a horizontal flat plate with constant heat flux, and aiding stagnation point flows about a horizontal flat plate with wall temperature varying as x^2. The governing parameter for mixed convection about horizontal surfaces in a porous medium is found to be $Ra/(RePr)^{3/2}$ as opposed to Gr/Re for mixed convection about inclined surfaces [11]. The criteria for pure and mixed convection about horizontal surfaces in porous media are established.

ANALYSIS

Consider the combined free and forced convection in a porous medium adjacent to a horizontal heated or cooled surface with assisting external flow $U_\infty(x)$ as shown in Fig. 1. In the mathematical formulation of the problem, we shall assume that (i) the convective fluid and the porous medium are everywhere in local thermodynamic equilibrium, (ii) the temperature of the fluid is everywhere below boiling point, (iii) properties of the fluid and the porous medium such as viscosity, thermal conductivity, specific heats, thermal expansion coefficient, and permeability are constant, and (iv) the Bousinesq approximation can be applied. Under these assumptions the governing equations are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u=-\frac{K}{\mu}\frac{\partial p}{\partial x}, \tag{2}$$

$$v=-\frac{K}{\mu}\left(\frac{\partial p}{\partial y}\pm\rho g\right), \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

$$\rho=\rho_\infty[1-\beta(T-T_\infty)], \tag{5}$$

where the "+" sign in equation (3) refers to the case of a heated impermeable surface facing upward [Figs. 1(a) and (b)] while the "−" sign refers to the case of a cooled impermeable surface facing downward [Figs. 1(c) and (d)]. In equations (1)–(5), u and v are the Darcy's velocities in the horizontal and vertical directions; ρ, μ and β are the density, viscosity, and the thermal expansion coefficient of the convecting fluid; K is the permeability of the porous medium; $\alpha \equiv k_m/(\rho_\infty C)_f$ is the equivalent thermal diffusivity

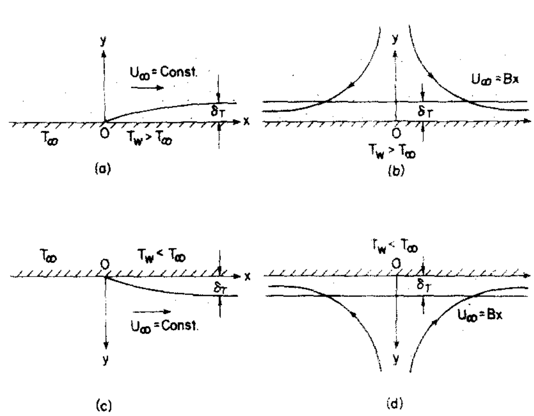

FIG. 1. Coordinate systems.

with $(\rho_\infty C)_f$ denoting the product of density and specific heat of the convecting fluid, and k_m the thermal conductivity of the saturated porous medium given by $k_m=(1-n)k_s+nk_f$ where n is the porosity of the medium, k_s and k_f are the thermal conductivity of the solid and the convecting fluid respectively; T, p and g are the temperature, pressure and the gravitational acceleration. The subscript "∞" refers to the condition at infinity.

The boundary conditions for the problem are

$$y=0,\quad T_w=T_\infty\pm Ax^\lambda,\quad v=0, \tag{6a,b}$$

$$y\to\infty,\quad T=T_\infty,\quad u=U_\infty(x), \tag{7a,b}$$

where $A>0$ and the "+" and "−" signs in equation (6a) are for a heated impermeable surface facing upward and for a cooled impermeable surface facing downward. Equation (6a) shows that the prescribed wall temperature is a power function of distance from the origin.

We now assume that (i) convection takes place in a thin layer adjacent to the heated or cooled surface, and (ii) outside this layer density of the fluid can be considered to be constant. Analogous to the classical boundary-layer theory, we shall separate the problem into two regions: the outer region where the fluid can be treated as incompressible and the inner region where density gradient exists and convection takes place. Thus, for the outer region, equations (2) and (3) can be written as

$$u=-\frac{\partial\phi}{\partial x}\quad\text{and}\quad v=-\frac{\partial\phi}{\partial y}, \tag{8}$$

where $\phi \equiv (K/\mu)(p\pm\rho g y)$ is the velocity potential. Substituting equation (8) into equation (1), we have

$$\nabla^2\phi=0, \tag{9a}$$

which is the Laplace equation for the outer region. Eliminating ϕ from equation (8) and with the resulting equation in terms of stream function ψ, we have

$$\nabla^2\psi=0, \tag{10}$$

where $u=\partial\psi/\partial y$ and $v=-(\partial\psi/\partial x)$. From potential flow theory, we know that the solution to equation (10) for flow over a horizontal surface, and stagnation point flow about a horizontal surface are $\psi=By$ and $\psi=Bxy$ which can be rewritten in a more compact form as $\psi = Bx^m y$ (and consequently $u = U_\infty = Bx^m$) with $m = 0$ and $m = 1$ for the two different external flow conditions.

We now turn our attention to the inner region, i.e. the boundary-layer region adjacent to the heated or cooled impermeable surface where density gradient exists. With boundary-layer approximations, equations (1)–(5) can be rewritten as [12]

$$\frac{\partial^2\psi}{\partial y^2} = \pm\frac{K\rho_\infty g\beta}{\mu}\frac{\partial T}{\partial x}, \tag{11}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\left(\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}\right). \tag{12}$$

Boundary conditions in terms of ψ are

$$y = 0, \quad T_w = T_\infty \pm Ax^\lambda, \quad \frac{\partial\psi}{\partial x} = 0, \qquad (13a,b)$$

$$y \to \infty, \quad T = T_\infty, \quad u = \frac{\partial\psi}{\partial y} = U_\infty = Bx^m, \qquad (14a,b)$$

where equation (14b) is the flow condition in the outer region with $m = 0$ for assisting flow over a horizontal flat plate at zero incident, and $m = 1$ for stagnation point flow about a horizontal impermeable surface.

To seek similarity solutions to equations (11) and (12) with boundary conditions (13) and (14), we introduce the following dimensionless variables

$$\eta = \left(\frac{U_\infty x}{\alpha}\right)^{1/2}\frac{y}{x}, \tag{15}$$

$$\psi = (\alpha U_\infty x)^{1/2} f(\eta), \tag{16}$$

$$\theta(\eta) = (T - T_\infty)/(T_w - T_\infty). \tag{17}$$

In terms of new variables, it can be shown that the velocity components are given by

$$u = U_\infty f'(\eta), \tag{18}$$

$$v = \tfrac{1}{2}\left(\frac{\alpha U_\infty}{x}\right)^{1/2}[(1-m)\eta f' - (1+m)f], \tag{19}$$

and the governing equations (11) and (12) become

$$f'' = -\frac{\rho_\infty g K\beta A}{\mu B}\left(\frac{\alpha}{B}\right)^{1/2} x^{(2\lambda-1-3m)/2}\left[\lambda\theta + \frac{m-1}{2}\eta\theta'\right], \tag{20}$$

$$\theta'' = \lambda\theta f' - \frac{1+m}{2} f\theta', \tag{21}$$

with boundary conditions given by

$$\eta = 0, \quad \theta = 1, \quad f = 0, \qquad (22a,b)$$

$$\eta \to \infty, \quad \theta = 0, \quad f' = 1. \qquad (23a,b)$$

It is apparent that equation (20)–(23) will be independent of x if the exponent of x in equation (20) vanishes, i.e.

$$\lambda = (3m+1)/2 \text{ or } m = (2\lambda - 1)/3. \tag{24}$$

Under this restricted condition, equation (20) and (21) in terms of m become

$$f'' = -\frac{Ra}{2(RePr)^{3/2}}[(3m+1)\theta + (m-1)\eta\theta'], \tag{25}$$

$$\theta'' = \tfrac{1}{2}[(3m+1)\theta f' - (m+1)f\theta'], \tag{26}$$

which can also be written in terms of λ to give

$$f'' = -\frac{Ra}{(RePr)^{3/2}}\left[\lambda\theta + \left(\frac{\lambda-2}{3}\right)\eta\theta'\right], \tag{27}$$

and

$$\theta'' = \lambda\theta f' - \left(\frac{\lambda+1}{3}\right)f\theta', \tag{28}$$

where

$$\frac{Ra}{(RePr)^{3/2}} \equiv \frac{\rho_\infty g\beta K|T_w - T_\infty|x/\mu\alpha}{\left(\frac{U_\infty x}{\alpha}\right)^{3/2}} = \frac{\rho_\infty g\beta K A}{\mu B}\left(\frac{\alpha}{B}\right)^{\frac{1}{2}}.$$

Equations (25) and (26) or equations (27) and (28) are the governing equations for mixed convection about horizontal impermeable surfaces in a porous medium where $m = 0$ and $\lambda = 1/2$ correspond to mixed flows over a horizontal flat plate with $T_w \alpha (x)^{\frac{1}{2}}$, while $m = 1$ and $\lambda = 2$ correspond to stagnation point flow with $T_w \alpha\, x^2$.

It is worth noting that the governing parameter for mixed horizontal boundary-layer flows is $Ra/(RePr)^{3/2}$, and that the limiting case of $Ra/(RePr)^{3/2} = 0$ corresponds to forced boundary-layer flows. Let's examine the limiting case of $Ra/(RePr)^{3/2} = 0$ in some detail. For this special case, equations (20)–(23) are independent of x for arbitrary values of m and λ. Furthermore, equation (20) can be integrated with the aid of equations (22b) and (23b) to give

$$f' = 1 \quad \text{and} \quad f = \eta. \qquad (29a,b)$$

Substituting equation (29) into equations (16), (18), (19) and (21) yields

$$\psi = Bx^m y, \tag{30}$$

$$u = U_\infty = Bx^m, \quad v = -Bx^{m-1}y, \qquad (31a,b)$$

and

$$\theta'' = \lambda\theta\,\frac{1+m}{2}\eta\theta'. \tag{32}$$

Equation (32) with equations (22a) and (23a) are the governing equation and boundary conditions for temperature distribution inside a thermal boundary layer of a forced flow in a porous medium.

RESULTS AND DISCUSSION

Equations (25) and (26) or equations (27) and (28) with boundary conditions (22) and (23) are integrated numerically by means of the Runge–Kutta method with a systematic guessing of $\theta'(0)$ and $f'(0)$ by the shooting technique. Numerical computations were carried out for aiding flows with the values of $Ra/(RePr)^{3/2}$ from 0 to 15. Results for $\theta(\eta)$ and $f'(\eta)$, for $\lambda = 1/2$ and $m = 0$ as well as for $\lambda = 2$ and $m = 1$ are presented in Figs. 2 and 3.

Of particular interest in geothermal applications are the heat-transfer rate and the thermal boundary-layer thickness. Consider first the local surface heat flux along the horizontal impermeable surface which is given by

$$q = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = kAx^{2m}\left(\frac{B}{\alpha}\right)^{1/2}[-\theta'(0)], \tag{33}$$

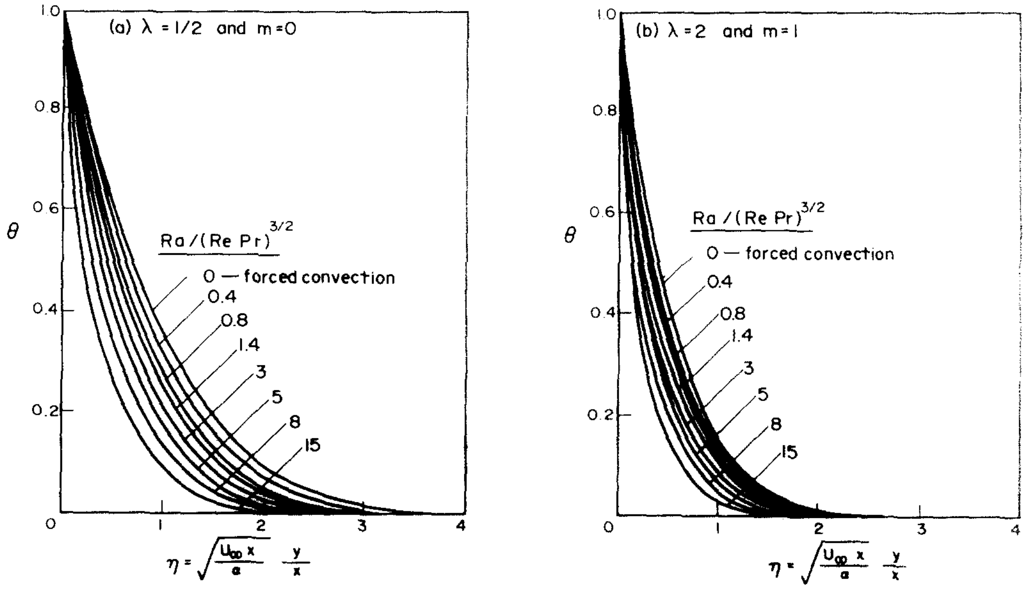

FIG. 2. Dimensionless temperature profiles for mixed convection with aiding external flows: (a) horizontal flat plate at zero angle of attack with constant heat flux ($m = 0$ and $\lambda = 1/2$), (b) stagnation point flow with $T_w \propto x^2$ ($m = 1$ and $\lambda = 2$).

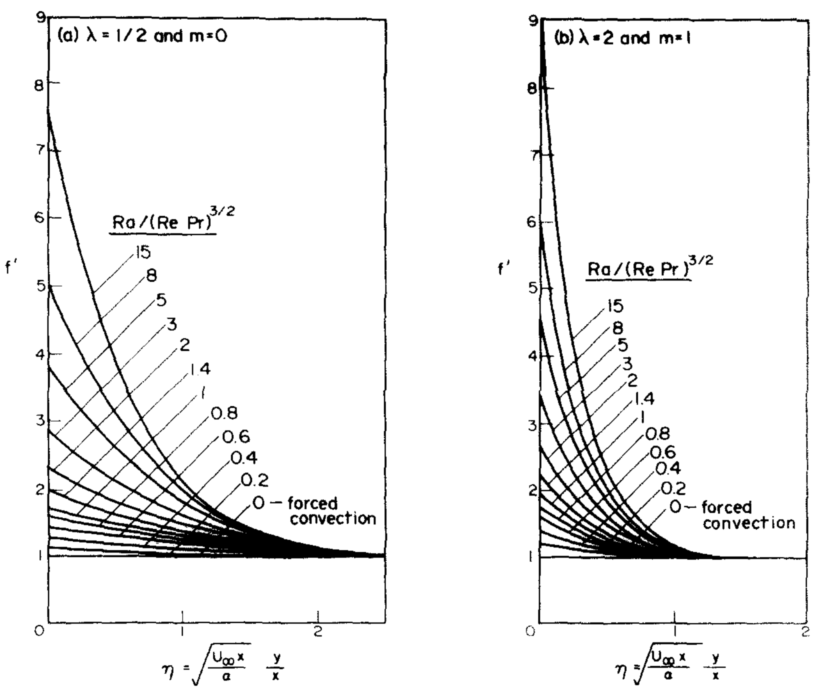

FIG. 3. Dimensionless velocity profiles for mixed convection with aiding external flows: (a) horizontal flat plate at zero angle of attack with constant heat flux ($m = 0$ and $\lambda = 1/2$), (b) stagnation point flow with $T_w \propto x^2$ ($m = 1$ and $\lambda = 2$).

Table 1. Values of $[-\theta'(0)]$, $f'(0)$ and η_T for aiding flows

	$m = 0$ and $\lambda = 1/2$			$m = 1$ and $\lambda = 2$		
$Ra/(RePr)^{3/2}$	$-\theta'(0)$	$f'(0)$	η_T	$-\theta'(0)$	$f'(0)$	η_T
0	0.8862	1.000	3.2	1.595	1.000	2.0
0.6	1.028	1.474	2.9	1.863	1.578	1.9
1.0	1.102	1.747	2.8	2.004	1.916	1.8
2.0	1.249	2.348	2.6	2.291	2.666	1.7
5.0	1.550	3.799	2.2	2.879	4.495	1.5
8.0	1.761	4.999	2.0	3.292	6.010	1.3
15.0	2.113	7.345	1.7	3.982	8.980	1.2

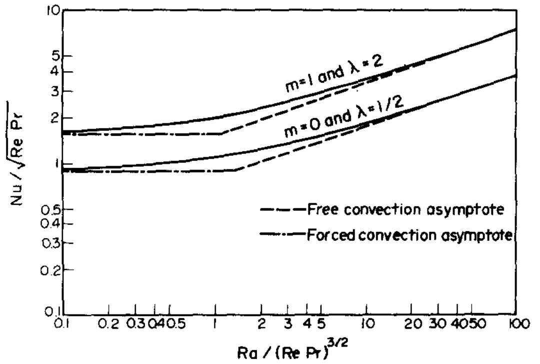

FIG. 4. Heat-transfer results for mixed convection with aiding external flows: (a) horizontal flat plate at zero angle of attack with constant heat flux ($m = 0$ and $\lambda = 1/2$) and (b) stagnation point flow with $T_w \alpha x^2$ ($m = 1$ and $\lambda = 2$).

where the values of $[-\theta'(0)]$ for $m = 0$ and $m = 1$ at selected values of $Ra/(RePr)^{3/2}$ are presented in Table 1. Equation (33) shows that local surface heat flux is constant for $m = 0$. Equating equation (33) to the definition of local heat transfer coefficient, i.e. $q = h(T_w - T_\infty)$, we have

$$\frac{Nu}{(RePr)^{1/2}} = [-\theta'(0)], \tag{34}$$

where $Nu \equiv hx/k_m$. Equation (34) for $m = 0$ and $m = 1$ is plotted in Fig. 4 as a function of $Ra/(RePr)^{1/3}$. The limiting cases of pure free convection and pure forced convection can be shown as asymptotes in the same figure. According to equation (34) and Table 1, the expressions for pure forced convection [where $Ra/(RePr)^{3/2} = 0$] are

$$\frac{Nu}{(RePr)^{1/2}} = 0.8862, \text{ for } m = 0 \text{ and } \lambda = 1/2, \tag{35a}$$

$$\frac{Nu}{(RePr)^{1/2}} = 1.595, \text{ for } m = 1 \text{ and } \lambda = 2. \tag{35b}$$

The corresponding expressions for pure free convection about a horizontal impermeable surface embedded in a porous medium are [12]

$$\frac{Nu}{(Ra)^{1/3}} = 0.8164, \text{ for } \lambda = 1/2,$$

$$\frac{Nu}{(Ra)^{1/3}} = 1.571, \text{ for } \lambda = 2,$$

which can be rewritten as

$$\frac{Nu}{(RePr)^{1/2}} = 0.8164\left[\frac{Ra}{(RePr)^{3/2}}\right]^{1/3}, \text{ for } \lambda = 1/2, \tag{36a}$$

$$\frac{Nu}{(RePr)^{1/2}} = 1.571\left[\frac{Ra}{(RePr)^{3/2}}\right]^{1/3}, \text{ for } \lambda = 2, \tag{36b}$$

It is shown in Fig. 4 that equation (34) approaches the forced and free convection limits [given by equations (35) and (36) respectively] as the values of $Ra/(RePr)^{3/2}$ approach zero and infinity. The criteria for pure or mixed convection about a horizontal surface in a porous medium can be established if the 5% deviation rule [2] is applied. It follows that

$0 < Ra/(RePr)^{3/2} < 0.16$ forced convection
$0.16 < Ra/(RePr)^{3/2} < 5$ mixed convection
$15 < Ra/(RePr)^{3/2} < 0$ free convection.

Consider next the expression for thermal boundary-layer thickness. If η_T is the value of η at the edge of the thermal boundary layer, i.e. where $\theta(\eta)$ has a value of 0.01, we have,

$$\frac{\delta_T}{x} = \frac{\eta_T}{(RePr)^{1/2}}, \tag{37}$$

where the values of η_T for $m = 0$ and $\lambda = 1/2$ as well as $m = 1$ and $\lambda = 2$ at selected values of $Ra/(RePr)^{3/2}$ are presented in Table 1. It is worth noting that for stagnation point flow ($m = 1$ and $\lambda = 2$), equation (37) reduces to $\delta_T = (\alpha/B)^{1/2}\eta_T$ which is independent of x. It will be of interest to show the values of $(RePr)^{1/2}\delta_T/x$ in the free and forced convection limits. This is done in Fig. 5 where the free convection asymptotes are given by Cheng and Chang [12]

$$(RePr)^{1/2}\delta_T/x = \frac{5.0}{Ra/(RePr)^{3/2}} \quad (\lambda = 1/2), \tag{38a}$$

$$(RePr)^{1/2}\delta_T/x = \frac{3.7}{Ra/(RePr)^{3/2}} \quad (\lambda = 2). \tag{38b}$$

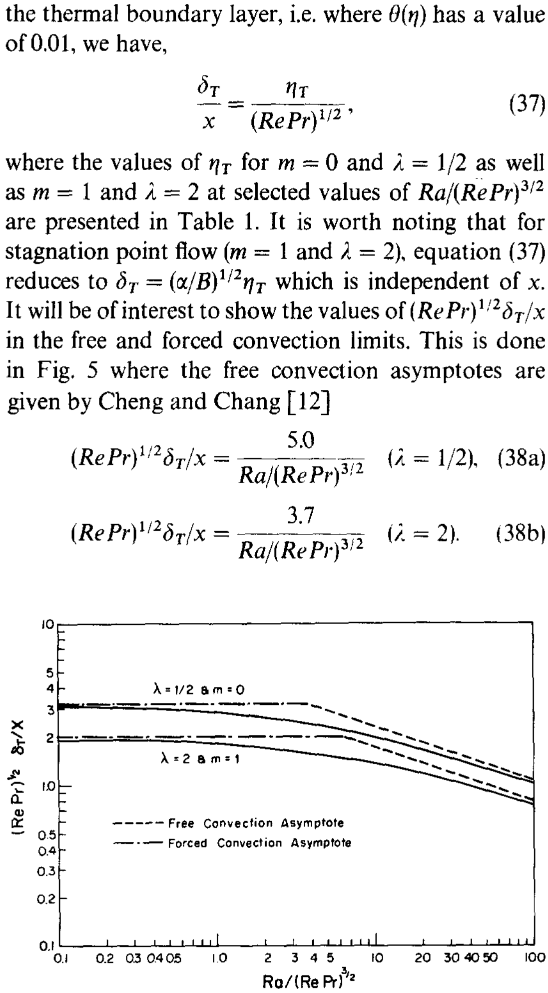

FIG. 5. Dimensionless boundary-layer thickness parameter for mixed convection about a horizontal flat plate.

CONCLUDING REMARKS

An analysis has been made for mixed convection in horizontal boundary layer flows in a saturated porous medium with aiding external flows (i.e. $B > 0$). It is found that the governing parameter for the problem is $Ra/(RePr)^{3/2}$ as opposed to Gr/Re which is the governing parameter for mixed convection about inclined plates in a porous medium [11]. Similarity solutions have been obtained for (i) mixed convection about a horizontal flat plate at zero angle of attack with constant heat flux, and (ii) mixed convection in stagnation point flows about a horizontal flat plate with $T_w \alpha x^2$. It is also found that no similarity solution is possible for mixed convection in horizontal boundary-layer flows in a porous medium with opposing external flows (i.e. $B < 0$).

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SOLUTIONS EN SIMILITUDE DE LA CONVECTION MIXTE POUR DES SURFACES IMPERMEABLES HORIZONTALES DANS DES MILIEUX POREUX SATURES

Résumé—On conduit l'analyse de couche limite pour la convection mixte autour d'une plaque plane et horizontale dans un milieu poreux saturé et avec des écoulements externes favorables: on a trouvé que le paramètre fondamental est $Ra/(RePr)^{3/2}$. Des solutions en similitude sont obtenues pour (1) des plaques planes horizontales à angle d'attaque nul et à flux de chaleur constant, pour (2) des écoulements de point d'arrêt autour de plaques planes horizontales dont la température de surface varie proportionnellement à x^2. Des profils de température et de vitesse sont présentés dans ces deux cas pour des valeurs particulières de $Ra/(RePr)^{3/2}$. Le transfert thermique approche asymptotiquement celui de convection forcée ou libre lorsque $Ra/(RePr)^{3/2}$ tend vers les limites 0 ou ∞. On etablit les critères pour la convection forcée ou mixte autour des plaques planes dans les milieux poreux.

AEHNLICHKEITSLÖSUNGEN FÜR DIE GEMISCHTE KONVEKTION ÜBER EINER HORIZONTALEN, UNDURCHLÄSSIGEN OBERFLÄCHE IN EINEM GESÄTTIGTEN PORÖSEN MEDIUM

Zusammenfassung—Für die gemischte Konvektion über einer horizontalen, ebenen Platte in einem gesättigten porösen Medium mit zusätzlichen, von außen aufgebrachten Strömungen wird eine Grenzschichtuntersuchung durchgeführt. Als bestimmender Parameter ergibt sich $Ra/(RePr)^{3/2}$. Aehnlichkeitslösungen werden erhalten (1) für horizontale, längsangeströmte, ebene Platten mit konstanter Wärmestromdichte und (2) für Staupunktströmungen, um horizontale, ebene Platten mit einer Wandtemperaturverteilung $T_w \sim x^2$. Zu beiden Fällen werden für ausgewählte Werte von $Ra/(RePr)^{3/2}$ Temperatur- und Geschwindigkeitsprofile angegeben. Für $Ra/(RePr)^{3/2}$ gegen 0 bzw. ∞ nähert sich der Wärmeübergang asymptotisch den Werten für die erzwungene bzw. freie Konvektion. Es werden Kriterien für reine und gemischte Konvektion über horizontalen, ebenen Platten in porösen Medien aufgestellt.

АВТОМОДЕЛЬНЫЕ РЕШЕНИЯ ДЛЯ СЛУЧАЯ СМЕШАННОЙ КОНВЕКЦИИ ОТ ГОРИЗОНТАЛЬНЫХ НЕПРОНИЦАЕМЫХ ПЛАСТИН В НАСЫЩЕННЫХ ПОРИСТЫХ СРЕДАХ

Аннотация — Проведено исследование пограничного слоя при смешанной конвекции у горизонтальной плоской пластины в насыщенной пористой среде при наличии спутного внешнего течения, для которого определяющим является параметр $Ra/(RePr)^{3/2}$. Получены аналитические решения для горизонтальных плоских пластин при нулевом угле атаки и постоянном подводе тепла и для застойных течений у горизонтальных плоских пластин с температурой стенки, изменяющейся как $T_w \alpha X^2$. Представлены температурные и скоростные профили для этих двух случаев при выбранных значениях $Ra/(RePr)^{3/2}$. Показано, что скорость переноса тепла асимптотически приближается к значениям при свободной и вынужденной конвекции по мере того как $Ra/(RePr)^{3/2}$ стремится к пределам 0 и ∞. Установлены критерии для случая чистой и смешанной конвекции у горизонтальных плоских пластин в пористых средах.